

HOW TO INTERPRET, UNDERSTAND, AND PREDICT STRATAL GEOMETRIES USING STRATAL-CONTROL SPACES AND STRATAL-CONTROL-SPACE TRAJECTORIES

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Abstract

Interpreting and predicting basin-margin stratal geometries requires understanding of controls such as variations in supply and accommodation, ideally based upon independent quantitative evidence. Stratal-control spaces are a new tool to analyze controls on strata. A stratal-control space is an area, volume, or perhaps a higher-dimensional space, defined by a range of values of the controlling processes subsidence, sediment supply, and eustasy. A three-dimensional stratal-control volume with axes of subsidence, sediment supply, and eustatic rates of change can be populated with probabilities derived from analysis of time series of subsidence, supply and eustasy. These empirical or theoretical probabilities indicate the likelihood of occurrence of any particular combination of control rates defined by any point in the volume. The stratal-control volume can then be analyzed to determine which parts of the volume represent relative sea-level fall and rise, where in the volume particular stacking patterns will occur, and how probable those stacking patterns are. For outcrop and subsurface analysis, using a stratal-control *area* with eustasy and subsidence combined on a relative sea-level axis allows similar analysis, and may be preferable. A stratal-control trajectory is a history of supply and accommodation rates, interpreted from outcrop or subsurface data, or observed in analogue and numerical experiments, and plotted as a series of linked points forming a trajectory through a stratal-control space. Two theoretical and one actual outcrop example are presented to demonstrate how stratal-control trajectories can be analyzed to determine which controls are dominant. The accommodation supply trajectory range ratio (ASTRR) is a useful metric to characterise trajectory geometry. Trajectories with $ASTRR > 1$ can be considered accommodation-dominated, and $ASTRR < 1$ indicates a supply-dominated trajectory. Calculating the range of stratal-control probabilities along the trajectory indicates the probability of the rates of change of subsidence, supply, and eustasy required to form the interpreted stratal geometry. Both types of stratal-control-trajectory analyses can provide important additional understanding and prediction of how, why, and where stratal geometries form.

Introduction

Interpreting and predicting basin-margin stratal geometries requires an understanding of how supply, accommodation, and a number of other factors vary to control these geometries (Schlager, 1993; Helland-Hansen and Gjelberg, 1994; Catuneanu and Zecchin, 2013; Burgess and Prince, 2015). Based on this understanding, a description of strata following standard sequence stratigraphic methods (Catuneanu, 2006; Neal et al., 2016) can form the basis for interpretations of genetic mechanisms that allow prediction of elements not directly observed. Example predictions are volume of sediment bypass on a stratigraphic surface, the likely three-dimensional geometry of a stratal unit observed in only two dimensions, and the probable lithology of a seismic package located down depositional dip of available well control. Catuneanu and Zecchin (2016a) argued that a "statistical norm" indicates that it is reasonable to assume that accommodation variations are a prevalent control, for example on sequence-boundary formation. Burgess and Prince (2016) questioned whether there is sufficient evidence to support this claim.

It is important that interpretations of stratal architectures reflect realistic magnitudes and rates of change of accommodation and supply (Heller et al. 1993). Ideally, interpretations of strata should also provide the frequency with which changes of a particular magnitude occur. In other words, if sequence stratigraphic interpretations of strata across a range of Phanerozoic ages require a magnitude of relative sea-level change that occurs only very rarely during that time interval, that would suggest a possible problem with the interpretations. Stratigraphic solution sets offer a method to provide the required evidence (Heller et al., 1993; Hampson, 2016). They consist of a three-dimensional plot of rate of change of eustasy, subsidence, and sediment supply. The range of rates of change required to generate a particular stratal architecture is plotted based on analysis using a simple but plausible geometric model of stratal response to the three controls. Limits of each axes are defined based on maximum rates calculated from the observed strata, and then a sub-volume is defined that represents the solution set that is the most likely combination of rates to

account for the observed stratal architecture. Despite their utility, application of stratigraphic solution sets has been surprisingly limited.

The work presented below defines a method that could provide that missing quantitative evidence to determine the relative importance of the multiple possible stratal controls. This work develops the solution-set method by defining and constructing stratal-control spaces, both volumes and areas, and stratal-control trajectories. These stratal-control spaces are different from solution sets because they include calculated probabilities that indicate what rates of eustasy, tectonic subsidence and uplift, and sediment supply are most likely to occur, as well as what this might mean for any particular stratigraphic interpretation.

Methods and Results

Matlab code that implements the method described in this work is given in Appendix 1.

Defining Probability Density Functions for Rates of Change of Subsidence, Sediment Supply, and Eustasy

Data sets composed of times series of tectonic, sediment-supply, and eustatic forcing can be analyzed to measure the frequency of occurrence of particular rates of change. For example, analysis of a well-constrained eustatic-sea-level curve can show how often a rate of 10 m of rise per 100 ky occurs, and analysis of a curve of subsidence and uplift can do the same. Meaningful analysis always requires definition of a practical time interval over which rates of change in accommodation and supply will be calculated. If the interval is too long, important information might be missed on how variable the rates of change have been, especially with respect to the high-frequency components. If the interval is too short it will be difficult to apply to ancient strata because the interval will be shorter than the resolution with which durations of stratal units can be determined. Also, the interval cannot be shorter than the resolution of the data, for example the 100 ky data-point

resolution in most of the eustatic curve. Given these constraints, 100 ky is a reasonable choice of time interval, so throughout this analysis rates of change have been calculated for 100 ky intervals. Carrying out this analysis for a range of processes of accommodation and supply, and across a range of values for each process allows construction of probability density functions (PDFs) that characterize variations of accommodation and supply through time (Fig. 1). Note that these PDFs are intended to illustrate how a stratal-control space can be constructed, populated, and analyzed. This study is intended to demonstrate how to construct and analyze stratal-control spaces. It is not intended to be a definitive statement about all aspects of how rates of accommodation and supply vary. Much further work is required to compile and analyze more definitive time series and PDFs to fully represent these variations.

Rates of Subsidence and Uplift

Tectonic subsidence and uplift occur by a variety of processes. To construct an initial PDF for subsidence and uplift, subsidence due to lithospheric stretching, subsidence and uplift due to flexural loading of the lithosphere, and subsidence and uplift driven by dynamic topography due to mantle processes have been analyzed. Subsidence curves for sediment-loaded lithospheric extension were calculated with beta values ranging from 1.5 to ∞ , each for a total duration of 220 My, the first 20 My representing a synrift phase (Fig. 3.16 from Allen and Allen 2005). Curves of flexural subsidence and uplift (Fig. 4.30b from Allen and Allen, 2005) were analyzed that represent four foreland-basin locations with increasing distance from the initial edge of an advancing tectonic load. Rates range from $\approx 1 \text{ km My}^{-1}$ of subsidence to $\approx 300 \text{ m My}^{-1}$ of uplift. Uplift and subsidence due to dynamic topography was calculated from the models of Moucha et al. (2008) showing uplift and subsidence histories from 0 to 30 Ma on the North American and West African Atlantic margins. Rates in this case are a maximum of $\approx 7 \text{ m My}^{-1}$ for subsidence and $\approx 12 \text{ m My}^{-1}$ for uplift. Frequency of occurrence of all these rates were merged into a single PDF converted to m per 100 ky (Fig. 2A).

Rates of Sediment Supply

Time-series data on sediment supply remain rare and uncertain compared to eustatic or relative-sea-level examples. Carvajal et al. (2009) provide a time series of relative magnitude (low to high) of sediment supply from the Paleogene of the Gulf of Mexico. Relative variation in supply magnitude through time is estimated from the thickness and area of slope and basin-floor fan strata. The time series represents a history of sediment supply that bypassed the shelf margin and therefore probably reflects a combination of different factors causing variation through time. Variations in topset accommodation are likely one such control, though these may have only relatively small effects on deep-water supply, especially in greenhouse times (Burgess and Hovius, 1998; Blum and Tornquist, 2000). This supply time series is therefore used here as an illustrative example only, to provide supply values per 100 ky interval and show how sediment supply might vary over geologic time. The times series gives only relative magnitudes through time, so to calculate actual values of supply magnitude the normalized low-to-high variation is converted to a volume rate in meters per 100 ky using the river-mouth supply rate from 24 modern rivers (Hovius, 1998; Burgess and Hovius, 1998). This generates 24 sediment-supply time series with the same variation through time but with different magnitudes (Fig. 1D). Combining the 24 scaled time series allows definition of a preliminary but still rather speculative example PDF for sediment supply rate (Fig. 2B).

Rate of Eustatic Rise and Fall

Miller et al. (2005) and Miller et al. (2011) used a combination of backstripping of continental-margin strata and analysis of $\delta^{18}\text{O}$ data to define a eustatic sea-level curve for the last 100 Ma. This curve is considered the best available currently, so it has been used here to analyse magnitudes of eustatic change for 100 ky intervals over the last 100 Ma. The oxygen-isotope-derived eustatic curve (Miller et al. 2011) comprises a value for eustatic sealevel each 1 ky derived by assuming that oxygen

isotope values are a reliable proxy for water temperature and therefore for continental ice sequestration and eustatic sealevel. For each 100 ky interval a starting, ending, minimum and maximum value for eustatic sealevel was determined. From this the maximum rise and maximum fall over each 100 ky interval was calculated, giving 9000 values of rate of rise and fall. These values were then binned at 10 m amplitude intervals from -100 m (negative values indicating a eustatic fall) to 100 m. A similar method was used to analyze the backstripped eustatic curve from Miller et al. (2005) but in this case the time increment on the curve was 100ky, so values for eustatic rise and fall could be calculated directly from the 885 successive values in the curve. These values were then binned in the same way. Binned frequencies from the oxygen isotope curve and the backstripped curve were then summed, and frequencies in each bin were divided by the total count of 9885 values to define a relative probability density function. (Fig. 2C).

What Is a Stratal-Control Space?

A stratal-control space is an area, volume, or perhaps a higher-dimensional space, defined by a range of values of processes that control stratal geometries. For example, a stratal-control volume is an example of a stratal-control space that has three dimensions, namely rates of change of subsidence, sediment supply and eustatic sea-level. Plotting these variables on x, y, and z axes with a specified range for each variable defines a stratal-control volume (Fig. 3). A stratal-control space can be populated with probability values derived from analysis of the controlling-variable time series. For example, a useful stratal-control volume could be populated with probabilities derived from analysis of time series of subsidence, supply and eustatic sea-level, either empirically or theoretically derived. These probabilities indicate likelihood of occurrence of any particular combination of control rates at any point in the volume. Similar spaces were used to define stratigraphic solution sets by Heller et al. (1993). While Heller et al. (1993) showed how the technique could be used to analyze a single outcrop case, here the technique is expanded to include controlling variable PDFs that define

probabilities of occurrence within the space and to define trajectories that can be used to describe how and why particular stratal configurations developed.

Defining and Classifying a Stratal-Control Volume

A stratal-control volume with axes of rates of subsidence, sediment supply, and eustatic sea level has been defined using the limiting range of values in the PDFs for subsidence, supply, and eustasy (Fig. 2). Analysis of the resulting stratal-control volume is useful for understanding how strata can form in response to the defined controls. The stratal-control volume can be divided into sub-volumes based on the processes that would operate and the stacking patterns that would occur in the volume given the relative values of the three controls (Fig. 3). A sub-volume (48.8% of the total volume) delineating relative sea-level fall is defined where eustasy and subsidence sum negative (units m Ky^{-1}), and a sub-volume of relative sea-level rise is defined where their sum is positive (51.2% of the total volume) (Fig. 3). Note that these volume proportions are not the same as the probabilities of occurrence given below.

A planar surface of constant relative sea level separates the falling- and rising-RSL volumes (Fig. 3). Subdividing the stratal-control volume according to predicted stacking patterns requires more detailed consideration of accommodation and supply. Accommodation is defined as the space available for sediment to fill between a depositional surface and the surface above which erosion will occur (Jervey 1988; Catuneanu 2006). However, Muto and Steel (2000) pointed out problems with this definition, and suggested instead "the thickness, measured at a specified site and time, of a space which becomes filled with sediments during a specified time interval". From this definition, accommodation can be measured only post deposition when the results of sedimentation are known, or with reference to some specific volume (e.g., a specified area of seafloor capped by sea level). The area of Holocene Mississippi delta topset and shelf deposition is $6.05 \times 10^{10} \text{m}^2$ (Blum and Roberts, 2009). Using this as an example area to define accommodation allows rate of creation of

accommodation (units $\text{m}^3 \text{ky}^{-1}$) and rate of sediment supply ($\text{m}^3 \text{ky}^{-1}$) to be compared in the stratal-control volume, allowing definition of a surface across which rate of supply changes from being more than rate of accommodation creation to less than the rate of accommodation creation. This surface separates conditions causing progradation (67.5% of the volume) from conditions causing aggradation or retrogradational stacking (32.5% of the volume). Consideration of proportions of total stratal-control volume that these subdivisions represent can help to determine what stacking patterns are more and less likely for a given topset area.

The sub-volumes can also be considered as volumes where accommodation or supply dominates. For example, the volume with falling relative sea level can be considered a part of the stratal-control volume where accommodation is likely the dominant control on stratal geometry via forced regression, though this is complicated by the different possible responses of depositional systems to this negative accommodation (Catuneanu 2006; Burgess and Prince 2015). The zone of transgression (Fig. 3) is also a zone where accommodation can be said to dominate, in the sense that rate of accommodation creation exceeds sediment supply and causes retrogradational stacking. In contrast the zone of unforced regression can be argued to be supply dominated. However, full consideration of accommodation-dominated versus supply-dominated stratal geometries also requires consideration of how the controls and therefore the stratal geometries change through time. This is considered further below.

Populating the Stratal-Control Volume with Probabilities

With the PDFs for rate of change of subsidence, supply and eustatic sea level (Fig. 1) it is possible to populate the stratal-control volume with probabilities. For example, one particular sub-volume or bin in the stratal-control volume represents a rate of eustatic change between 0 and 10 m per 100 ky, subsidence of 0 to 2.5 m per 100 ky and a sediment supply between $1.2 \times 10^{12} \text{ m}^3$ and $1.3 \times 10^{12} \text{ m}^3$ per 100 ky. The probability of this combination of control values can, like all other control value

combinations in the volume, be calculated from analysis of the time series for the controlling variables (Fig. 1). This is summarized in the defined PDFs (Fig. 2), and the resulting probability value can be recorded in this bin in the stratal-control volume.

Each PDF is sampled randomly to get a rate value for subsidence, eustasy, and sediment supply. Repeating this 10^6 times with a bootstrap resampling method calculates frequency of occurrence of values in the stratal-control volume that accurately reflects frequency of occurrence of each value in the PDF. This process defines probabilities for combined subsidence, sediment supply, and eustasy, which are then recorded and plotted in the stratal-control volume as binned values (Fig. 3). For example, the highest probability in the stratal-control volume defined here is 0.04 for eustatic change between 0 and 10 m per 100 ky, subsidence of 0 to 2.5 m per 100 ky, and a sediment supply between $1.2 \times 10^{12} \text{ m}^3$ and $1.3 \times 10^{12} \text{ m}^3$ per 100 ky. Note that even this maximum probability is rather low, less than 5%, because in the control time series analyzed the highest probability for a value of each control ranges from 0.45 to 0.21 (see highest values on the PDFs in Fig. 2), giving this an overall low combined probability in the volume.

With these probabilities calculated from the control time series and PDFs, it is possible to determine from the stratal-control volume the likelihood of occurrence of various control values, and of the combination of control values. Note that the probabilities are different from the dimensions of the volume or sub-volume within it that were discussed in the previous section; likelihoods of occurrence given here are calculated only from the magnitude of the probability values within the volume, not based on the dimensions of the volume as they were in the previous section. Based on the probabilities within the stratal-control volume the likelihood of relative sea-level fall and forced regression within this stratal-control volume is 0.45, whereas the probability of forcing that would produce rising relative sea-level is 0.55. Given the definition of accommodation above, the total probability of progradation in the volume, both forced and unforced, is 0.89. The probability of

retrogradation is 0.11. Within the sub-volume that represents relative sea-level rise, the probability of unforced regression is 0.66 and the probability of transgression is 0.34.

Further analyses are possible based on the probabilities in the control volume. Such analyses may lead to a better understanding of the relative importance of various controls on stratal architectures interpreted from outcrop and subsurface data. For example, with the volume of relative sea-level fall, high sediment supply may generally mean less probability of fluvial incision. Making explicit assumptions about how supply rate relates to sediment transport rate would allow calculation of the probability of fluvial incision versus topset aggradation (Muto et al. 2007; Prince and Burgess 2013) for this stratal-control volume. It may also prove instructive to pursue methods to compare these probabilities, or equivalent values calculated with different parameters, against the incidence of interpreted stacking patterns in the ancient record.

Stratal-Control-Volume Trajectories

Strata in any basin are the product of a history of variations in supply and accommodation through time, and probably also variation in other controls not so typically considered, e.g., sediment transport processes and rates and autogenic processes of various forms. Any history of rates of supply and accommodation rates (for example derived from shoreline and shelf-edge trajectories) can be plotted as a series of linked points forming a trajectory in a stratal-control space. Each trajectory runs from a starting point to an end point, representing an evolution of the rates of subsidence, supply and eustatic change (Fig. 3).

A stratal-control trajectory of this type can provide a useful quantitative graphical summary of how and why strata have evolved through time. For example, the orientation of the trajectory in the stratal-control volume (Fig. 3) provides one way to indicate which control or controls are likely to have been dominant through time, since large variations in rate of supply, subsidence, or eustatic

change drive concomitant stacking patterns. If a trajectory shows greatest variation, especially oscillatory variation, along the subsidence or eustasy axes, that would indicate an accommodation-dominated system (*sensu* Porebski and Steel 2006), as shown in the accommodation-dominated example defined in this control volume (Fig. 3). Conversely, if variation occurs mostly along the supply axis (Fig. 3) the trajectory could be considered supply dominated (*sensu* Porebski and Steel, 2006).

Each point along each trajectory falls in a part of the stratal-control volume with a particular probability of occurrence derived from the input PDFs, so any trajectory can be also analyzed in terms of these probabilities (Fig. 4). The accommodation dominated trajectory has a mean probability of 0.0015 for the volume it traverses, and individual trajectory point probabilities range from zero to ≈ 0.01 . The lower probabilities occur when the trajectory extends into areas of the volume with relatively high rates of eustatic change (Fig. 3), for example points 40 to 70 on the trajectory, all of which all have close to zero probabilities (Fig. 4). If this trajectory came from an interpretation of ancient strata, the probabilities could perhaps suggest that the control rates being invoked are not realistic. If it came from an analogue or numerical model, it might suggest that the chosen parameters are somewhat unrealistic. These probabilities can also be used to compare trajectories. The supply-dominated trajectory has a mean probability of 0.0017, slightly higher than the mean value for the accommodation-dominated trajectory. If two trajectories were equally able to explain an observed stratal geometry, the trajectory with the highest probability would be the more plausible interpretation given the particular control time series used as input for this control volume, though such a comparison may require robust tests of significance.

Stratal-control volumes and stratal-control-volume trajectories should be easy to construct from analogue and numerical experiments where generally all the necessary variables are known since they are inputs to the experiment. Constructing a stratal-control volume from outcropping or subsurface strata may be more difficult. With reasonable age and paleobathymetric control,

subsidence and uplift can be at least estimated from strata via backstripping analysis. However, reliable quantification of sediment supply rates and eustatic change are more difficult because eustasy remains a poorly known variable for much of the pre-Neogene geological record (e.g., Burton et al. 1987; Miall 2010) and the source-to-sink mass-balance process sediment supply history may not be easy to determine (e.g., Burgess and Hovius 1998; Martinsen et al. 2010). One way to help address this issue is to reduce the number of variables by combining eustasy and subsidence into relative sea level, in order to generate a 2D stratal-control area.

Stratal-Control Areas – a Method for Outcrop and Subsurface Analysis

Combining eustasy and subsidence into a single relative-sea-level variable reduces the stratal-control volume to a two-dimensional area (Fig. 5). All of the probabilities distributed on the eustasy and subsidence axes can be summed and projected into the 2D area. Most of the bins in the area have a low probability between 0 and 4×10^{-4} , with higher probabilities restricted to rates of relative sea-level rise of 20-40 m per 100 ky. The relative sea-level and stacking-pattern partitioning are similar to their equivalents in the 3D control volume, and the stratal-control trajectory can still be interpreted as accommodation or supply dominated, or some balance of these two (Fig. 4B). Note, however, that in this 2D case the supply-dominated trajectory has a higher mean probability of 0.0074 compared to 0.0051 for the accommodation-dominated trajectory.

One key advantage of this 2D control area plot is that it can be constructed more easily from outcrop or subsurface data, based on estimates of rates of relative sea-level change, without the need to isolate the tectonic and eustatic values. A second advantage is that the accommodation dominance or supply dominance of trajectories can be analyzed directly from the plot. If we assume that relative sea-level change is equivalent to either accommodation creation or destruction, then we can define a ratio of the accommodation trajectory range and the supply range such that

$$ASTRR = \frac{(a_{max}-a_{min})/A_{range}}{(s_{max}-s_{min})/S_{range}}$$

where a_{max} is the maximum accommodation creation rate on a trajectory, a_{min} is the minimum accommodation creation rate, A_{range} is the range of accommodation-creation values across the whole stratal-control area, s_{max} is the maximum supply rate on a trajectory, s_{min} is the minimum supply rate, and S_{range} is the range of supply values across the whole stratal-control area. Trajectories with $ASTRR > 1$ show greater variation across the range of accommodation values, and $ASTRR < 1$ examples show more variation across the range of supply values. For the two examples discussed (Fig. 5), the accommodation-dominated example has $ASTRR = 9.29$, and the supply-dominated example has $ASTRR = 0.38$, showing how the $ASTRR$ metric can distinguish between trajectory types and hence identify a dominant forcing factor.

Outcrop and Subsurface Data Example

Maastrichtian strata in the Great Divide and Washakie basins in southern Wyoming, USA, are arranged into a series of 15 clinothem, each defined by a bounding interval of partly transgressive, shale-prone strata (Carvajal and Steel 2006, 2012) (Fig. 6). The history of relative sea level and sediment supply reconstructed for these strata (Carvajal and Steel, 2012) is used here to define a stratal-control-area trajectory. Based on ammonite chronostratigraphy, the total duration of the clinoform strata is 1.5 - 1.9 My, so each clinothem is estimated to represent between 100 ky and 127 ky. Carvajal and Steel (2012) interpreted a shelf-edge trajectory, and this is analyzed to reconstruct a relative-sea-level history, based on the assumption that the clinoform rollover tracks relative sea level, and that topset aggradation occurs during rising relative sea level (Fig. 6). Note that the latter is a common but not particularly robust assumption (e.g., Swenson and Muto 2007; Petter and Muto 2008; Prince and Burgess 2013), but we use it here because it is a standard way to extract accommodation history (e.g., Neal et al. 2016). A sediment-supply history was determined by Carvajal and Steel (2012) using closely spaced log data (especially conductivity-log data) to correlate

strata and construct maps and volumes for each clinothem. Clinothem volumes were then separated into topset, slope, and basin-floor compartment volumes. Uncertainty in the duration of each clinoform is tackled by calculating rates of change based on an average duration.

A stratal-control-area trajectory for clinoforms 1 - 15 in Washakie Basin (Fig. 7) is determined by a combination of rate of sediment supply and rate of relative sea-level change. The trajectory has an ASTRR value of 72.91, showing that changes in rate of accommodation creation across the trajectory span a greater range than the changes in supply. The Washakie Basin stratal-control-area trajectory, despite its rather convoluted shape, clearly shows a contrast between an early-stage and late-stage development. Early-stage development reflects an irregular but overall rising rate of sediment supply as well as high but irregular rates of relative sea-level change (clinoforms 1 - 9). This early aggradational architecture was generated as clinoform amplitude increased, accompanied by extensive marine topsets (Carvajal and Steel 2012), consistent with basin deepening and high rates of relative sea-level change causing topset transgressions to penetrate far landward into the siliciclastic wedge. Late-stage parts of the trajectory (clinoforms 10 - 15), by contrast, reflect a consistently high sediment supply, but also a series of even larger changes in the rate of relative sea-level change. The late stages of the siliciclastic wedge were previously interpreted to reflect lower rates of aggradation and stronger progradation of nonmarine topsets across the shelf, as well as increased possibility of relative-sea-level falls (Carvajal and Steel, 2012). The stratal-control-area trajectory greatly improves our visualization and understanding of these changes in supply rate and relative-sea-level change rate, beyond what is possible to glean from the architectural diagram alone (Fig. 6).

Because the trajectory extends into parts of the control area with unusually high rates of relative-sea-level rise, the mean probability for the trajectory is 0.0052, so similar to the value for the supply-dominated-trajectory example, and higher than the accommodation-dominated example. Variation in the rate of relative-sea-level change on the trajectory (Fig. 7) spans a wide range of control area

(from -25 m to 140 m per 100 ky) while changes in sediment supply rate along the trajectory span a smaller proportion of the control area range, leading to an ASTRR value of 72.91. This strongly suggests that the trajectory and the strata from which it is derived should be considered to be accommodation controlled. Note however that this conclusion follows directly from the assumption that topset aggradation is controlled only by relative sea level; if this assumption was relaxed, other supply- and transport-dominated controls may appear more dominant (Swenson and Muto 2005; Prince and Burgess 2013). Note also that the conclusion that the Maastrichtian infill of the Washakie Basin was accommodation dominated does not contradict the earlier conclusion that the absolute sediment supply was unusually high (Carvajal and Steel 2006), but instead emphasizes the highly variable rates of subsidence and uplift that this Laramide tilted-block basin also experienced during infilling.

Discussion

Despite recognition that there are multiple controls on sequence development (e.g., Catuneanu and Zecchin 2016b) there is still a prevalence of interpretations that tend to focus on accommodation as a dominant control on formation of sequence boundaries and systems tracts (e.g., Posamentier and Vail 1988; Neal and Abreu 2009; Catuneanu and Zecchin 2016b). This focus does not mean that future interpretations of stratal geometries should also focus on accommodation as a dominant control, not least because without independent evidence the conclusion of accommodation control often stems only from an original assumption of accommodation control (Burgess and Prince 2016). Rather than make *a priori* assumptions, it would be more helpful to have independent, quantitative evidence to guide what is and what is not a reasonable interpretation of controls on a stratal geometry, a point originally addressed by Heller et al. (1993).

Definition of stratal-control spaces, populated with probabilities derived from analysis of the controlling-variable time series, and definition of control-space trajectories with associated probabilities of occurrence, provides a new and potentially effective method to generate

independent evidence to help assess whether an interpretation of strata is reasonable. These methods can also provide quantitative evidence to indicate which, if any, control dominates in a particular case. Stratal-control-space trajectories can significantly enhance visualization of the history of sediment supply change and relative-sea-level change that control outcrop strata, by adding information not discernible from more traditional plots such as architecture cross sections. Understanding which control is dominant, or indeed understanding that no single control dominates, can be the seed point for much useful prediction (e.g., the probability and possible volume of distal sand-prone strata).

Even the preliminary, simple cases and the single outcrop example presented here raise an interesting point. Describing a depositional system as accommodation driven or supply driven (e.g., Porebski and Steel, 2006) typically refers to a single value of the accommodation supply (AS) ratio e.g., a high-supply system. In contrast the original meaning related to temporal variations (cf. “In a Q-dominated mode of sedimentation, variations of sediment supply are far more dynamic than variations of any other regime variable”, Thorne and Swift 1991). The examples of analysis of stratal-control trajectories presented here support the view that a system should be classified as supply or accommodation dominated, based less on their absolute magnitude and more on how those variables change through time. The ASTRR metric provides a quantitative basis for such classification.

The method presented here needs to be applied and tested further on examples of outcrop and subsurface data interpretation, as well as examples from analogue and numerical experiments. Comparing control trajectories from interpretation of outcrop and subsurface data with trajectories defined from analogue and numerical experiments may be particularly useful in facilitating better communication between experimental and observational studies. Analysis of examples of outcrop and subsurface data where multiple possible controls can be invoked should also be particularly fruitful. For example, the method could perhaps prove useful in helping to understand tectonically

versus climatically forced sequence boundaries, or allogenic versus autogenic interpretations of strata, or in analysis of non-uniqueness and stratal symmetry (Burgess and Prince, 2015).

It is important to note that the controlling-factor time series used in this analysis are far from complete; what is presented here is a first step to demonstrate how this new method can be applied. Further development of these time-series data is required. For example, the eustatic curve from Miller et al. (2005) only extends to 100 Ma, so addition of a time series for eustasy before 100 Ma would be useful, with careful consideration of whether the eustatic curves adequately capture any high-frequency high-amplitude eustatic variations that may have occurred. Additional processes and examples of tectonic subsidence need to be added, perhaps with examples from backstripping studies that represent actual rather than theoretical subsidence histories. Discrete subsidence and uplift PDFs for various tectonic settings are probably the best way to proceed, for example with discrete PDFs for the various passive and active tectonic settings. Sediment supply histories are perhaps the least well known of the three controlling factors, so compilation of data on variations in sediment supply through time from a variety of tectonic and climatic settings is now particularly important (e.g., following the methods defined in Petter et al. 2013). A key question in this case will be where in the transport system should supply rate be measured? Obvious candidates include the river mouth in modern systems, and clinoform slope breaks in ancient strata, including the shelf-edge break of slope. That said, many other possibilities exist, their utility depending on how the results from analyses of stratal-control spaces will be applied.

Representation of a stratal-control space as a two-dimensional area makes it easier to apply in outcrop and subsurface data interpretations. However, there is actually no particular reason to assume that strata are controlled only by accommodation and supply. Other controls may be equally important, either allogenic or autogenic, or both (Burgess et al. 2006, Williams et al. 2011, Muto et al. 2016). This raises the possibility of more dimensions for a control space, not fewer. Substantial progress in understanding the complexities of how and why strata develop with particular

geometries and properties may require analysis of stratal-control volumes with more than three dimensions, similar to the phase spaces used in several areas of physics, and visualized and analyzed in innovative new ways.

Conclusions

1. Problems currently exist with interpretation of stratal geometries when a conclusion of accommodation control follows in a circular manner from an initial implicit assumption of accommodation control. Independent quantitative evidence to break this logic circle should prove very useful in many stratigraphic interpretations.
2. Probability density functions can be constructed from controlling-variable time series to quantify the probability of occurrence of rates of subsidence, sediment supply, and eustasy.
3. Stratal-control spaces can be defined and populated from the controlling-variable probability density functions. Control volumes and areas can be analyzed to determine where in the space accommodation or supply is dominant, where in the space observed stacking patterns might occur, and how probable those stacking patterns are based on the controlling-variable probability density functions.
4. Stratal-control trajectories can be defined from outcrop, subsurface and experimental data, and analyzed to check the plausibility of interpretations, and to integrate interpretations from outcrop and subsurface data with results from analogue and numerical experiments. The trajectory accommodation and supply range ratio (ASTRR) is a useful metric to characterize trajectory geometry. Trajectories with $ASTRR > 1$ can be considered accommodation dominated, with greatest variation across the range of accommodation values, and $ASTRR < 1$ indicates supply-dominated examples, with more variation across the range of supply values.
5. Stratal-control-area trajectories can significantly enhance visualization of changes in rate of supply and relative sea level controlling outcrop strata, by adding information that is not discernible from more traditional plots such as architecture cross sections.
6. Analysis of stratal-control spaces and control trajectories could be expanded beyond three dimensions to begin to more fully address stratal complexity.

7. There is much work to be done to better define the subsidence, sediment-supply, and eustasy time series that constitute the input to this method. More detailed consideration of various tectonic settings and processes, and analysis of sediment supply histories at various points in various different sediment-routing systems, would be a good way to start.

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Figure 1. Time series of tectonic subsidence, sediment supply, and eustasy used to construct and populate a stratal-control volume. **A)** Syn- and post-rift tectonic-subsidence curves for beta values ranging from 1.5 to infinity, the latter being the predicted subsidence for oceanic lithosphere. From Fig. 3.16 of Allen and Allen, 2005. **B)** Four subsidence and uplift curves modelled at different positions in a foreland basin. Each curve records water-loaded subsidence due to flexural isostasy of the lithosphere as an orogenic load migrates laterally towards and over the foreland basin. From figure 4.30b of Allen and Allen, 2005. **C)** Two curves showing subsidence and uplift driven by dynamic topography calculated from mantle-flow simulations (Moucha et al., 2008) **D)** Two examples showing how the normalized time series of relative sediment-supply volume, estimated from slope and basin-floor Paleogene strata of the Gulf of Mexico (Caravajal et al., 2009), have been scaled to give meters cubed per 100 ky using data on river mouth supply rate for 24 rivers from Hovius (1998). One time series is scaled by the river-mouth supply from the Mississippi, and the second is scaled with the Ebro river-mouth supply. The shape of each curve is identical, only the magnitudes differ. **E)** Time series of eustatic sea level time from Miller et al. (2005) and Miller et al. (2011). Before 10 Ma the curve is derived from backstripping analysis of US East Coast continental-margin strata. From 10 to 0 Ma the curve is derived from analysis of $\delta^{18}\text{O}$ data and forms a history of high-frequency glacioeustasy.

Figure 2. Estimates of the probability density functions (PDFs) for time series of rates of change in the subsidence, sediment supply and eustatic sea level shown in Figure 1. **A)** Relative frequencies of rates of tectonic subsidence and uplift calculated from the time series in Figure 1 A, B, and C. Positive values represent subsidence, and negative values represent uplift. **B)** Relative frequencies of sediment supply rate calculated from 24 sediment-supply time series, two of which are shown in Figure 1D. **C)** Relative frequency of rate of eustatic sea-level change calculated from the two eustatic sealevel time series in Figure 1E.

Figure 3. A stratal-control volume, showing the range of values for rate of subsidence and uplift, rate of sediment supply, and rate of eustatic sea-level change derived from the time series in Figure 1. The volume is divided into zones of forced regression, unforced regression, and transgression by 2D planes in three dimensions representing the positions in the volume where the rate of relative sea-level change is zero, and the position where (with assumptions about the area of accommodation) the controls would lead to aggradational stacking. Also plotted in the volume are probabilities of occurrence of these rates of change, so that the size of the cuboid plotted at each point in the volume indicates the probability of occurrence of that particular combination of rates of change for each control. Larger cuboids indicate a more probable rate. The highest probability of occurrence is 0.04 for the bin sub-volume with eustatic change between 0 and 10 m per 100 ky, subsidence of 0 to 2.5 m per 100 ky and a sediment supply between $1.2 \times 10^{12} \text{ m}^3$ and $1.3 \times 10^{12} \text{ m}^3$ per 100ky. The final elements on the plot are stratal-control trajectories. Two examples are plotted, representing hypothetical stratal geometries and evolutions, one accommodation dominated (blue line) and one supply dominated (red line). In each case the trajectory runs from a start point (circle) to an end point (cross), and crosses between different parts of the volume indicating changes from transgression to regression and from aggradation to progradation, and vice versa.

Figure 4. A) Strata control probabilities extracted from the 3D control volume along the two hypothetical stratal-control trajectories (Fig. 3). Blue bars are probabilities from the accommodation-dominated trajectory. Red bars are from the supply-dominated trajectory. Note that the trajectories have different lengths and so are composed of different numbers of points. The x-axis is effectively geological time, but since no ages are defined for the two hypothetical trajectory examples, here it is just labelled as point number or distance along the trajectory. The plot allows analysis and comparison of probabilities for trajectories. The supply-dominated trajectory has a mean probability

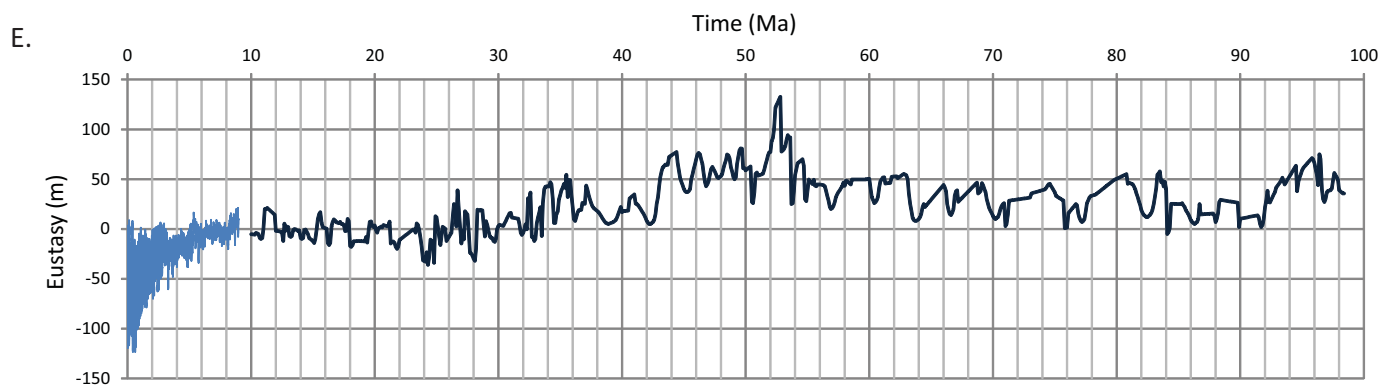
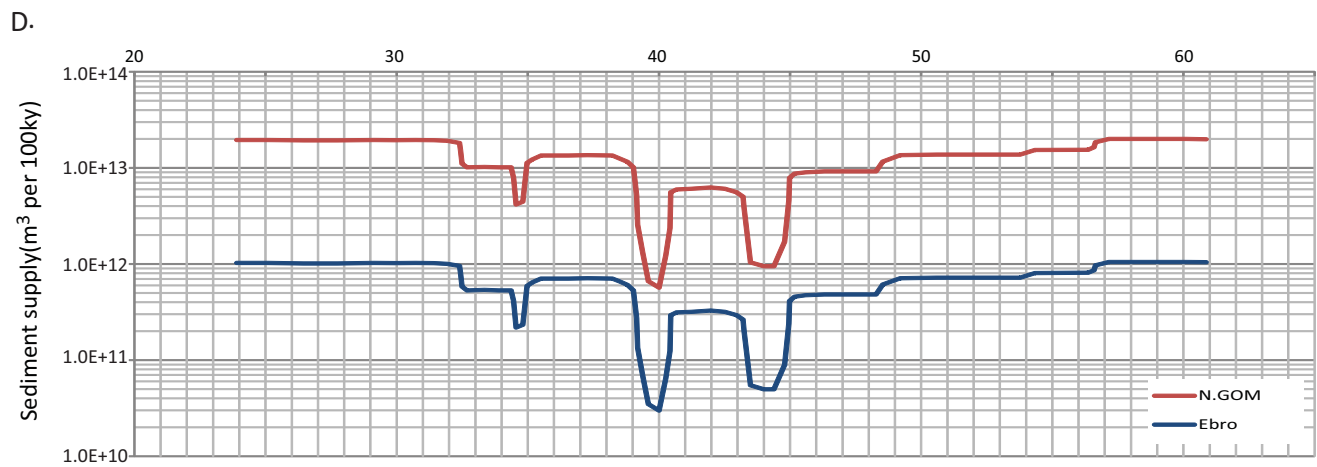
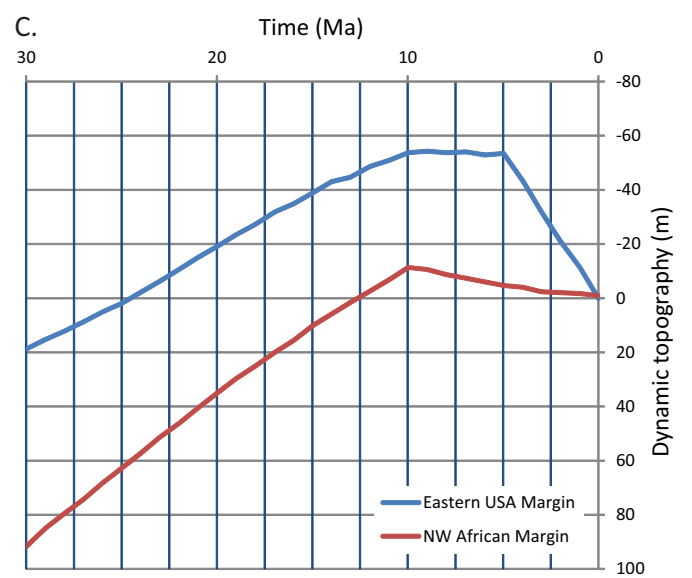
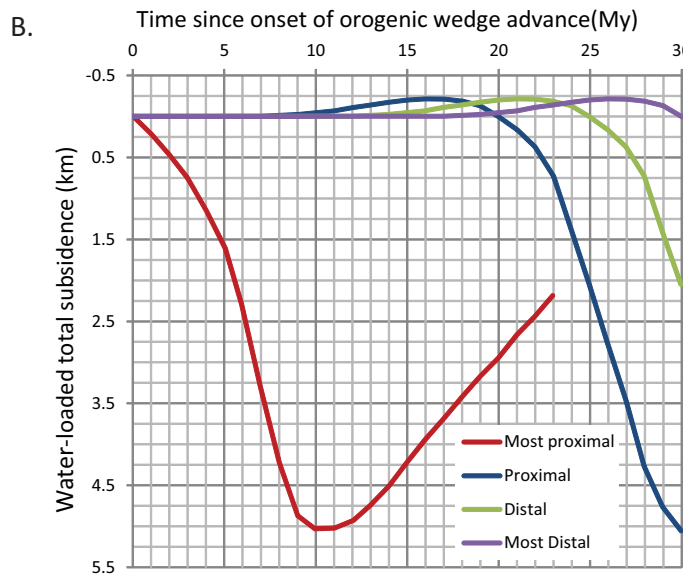
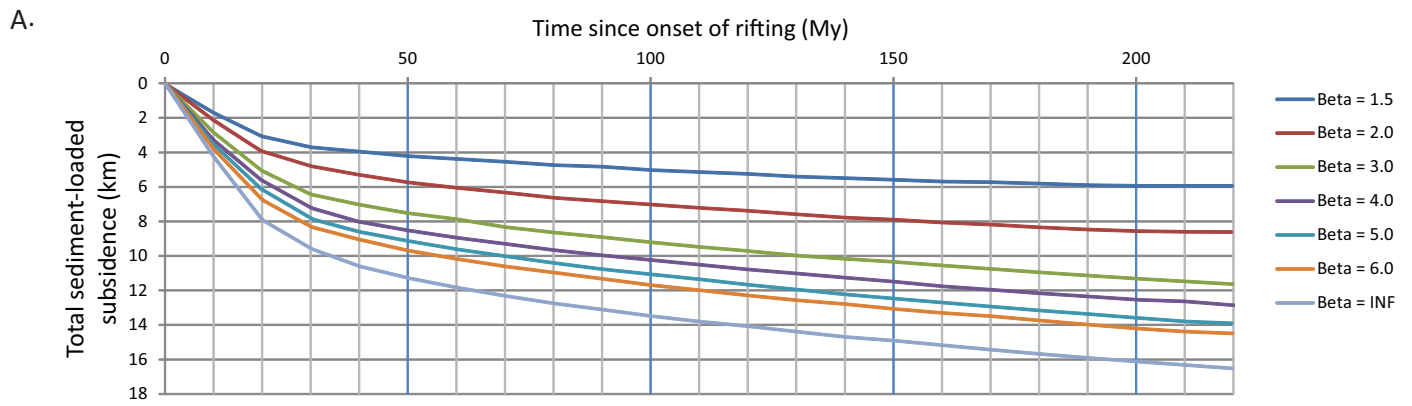
of 0.0017 with relatively high probabilities initially, but then moves into an area of the control volume with rates that do not occur frequently in the control time series (Fig. 1) and PDFs (Fig. 2). The accommodation-dominated trajectory has a lower mean probability of 0.0015, and because high rates of eustatic sea-level change are rare (Fig. 2C), the probabilities are low for points 1 to 20 and points 40 upwards, but are higher in the second quarter of the trajectory, when the rates of eustatic change are lower. **B)** Equivalent stratal-control probabilities extracted from the 2D control area (see Fig. 5) along the two hypothetical stratal-control trajectories and, in this case, also probabilities from the outcrop-example stratal-control trajectory (green bars). The probability values in the 2D area differ from those in the volume, because of the transformation from 3D to 2D with an accommodation-dominated mean of 0.0051 and a supply-dominated mean of 0.0074. However, the same pattern of variation through time is present for both trajectories in the 2D compared to 3D cases. The outcrop-derived trajectory points have probabilities generally similar to the two hypothetical examples with a mean of 0.0052, similar to the accommodation-dominated example.

Figure 5. A stratal-control area that is a 2D representation of the 3D control volume shown in Figure 3. Rates of eustatic sea-level change and subsidence have been combined into rate of relative sea-level change to effect a transformation to 2D. All of the same elements from the 3D volume are plotted here in 2D and can be interpreted in the same way. Note that white areas of the plot have probabilities of zero, representing rates of change of controls that do not occur in the input time series. The accommodation-dominated trajectory has $ASTRR = 9.29$, and the supply-dominated example has $ASTRR = 0.38$, so the label for each trajectory is appropriate.

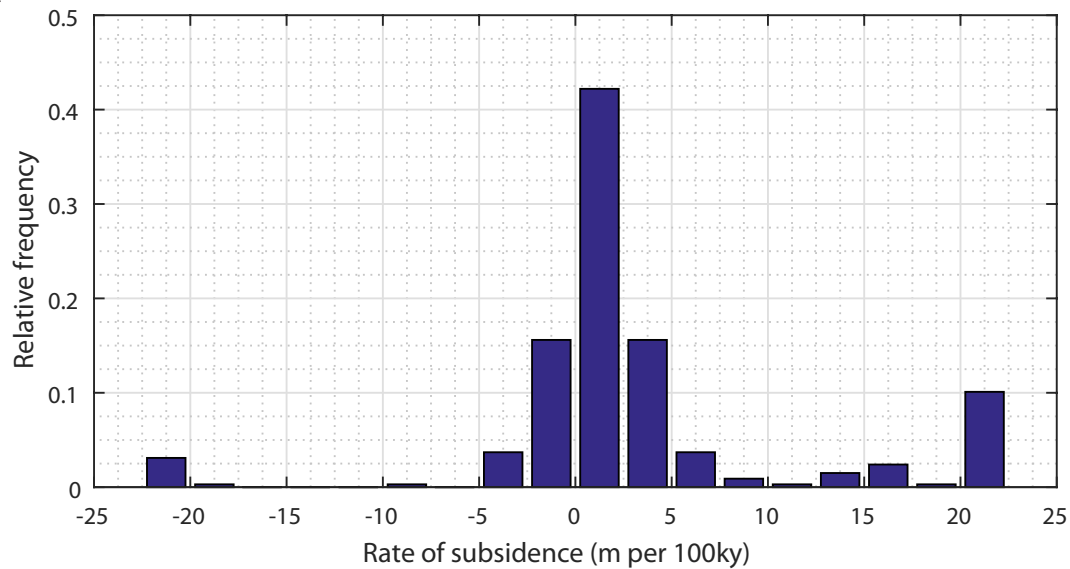
Figure 6. Cross section from the Lewis-Fox Hills-Lance system in the Washakie and Great Divide basins (Southern Wyoming) showing an accreting shelf margin composed of at least 17 clinothem. A

shelf-margin trajectory has been defined through the break-of-slope point on each clinoform. Note that the cross section has approximately x50 vertical exaggeration.

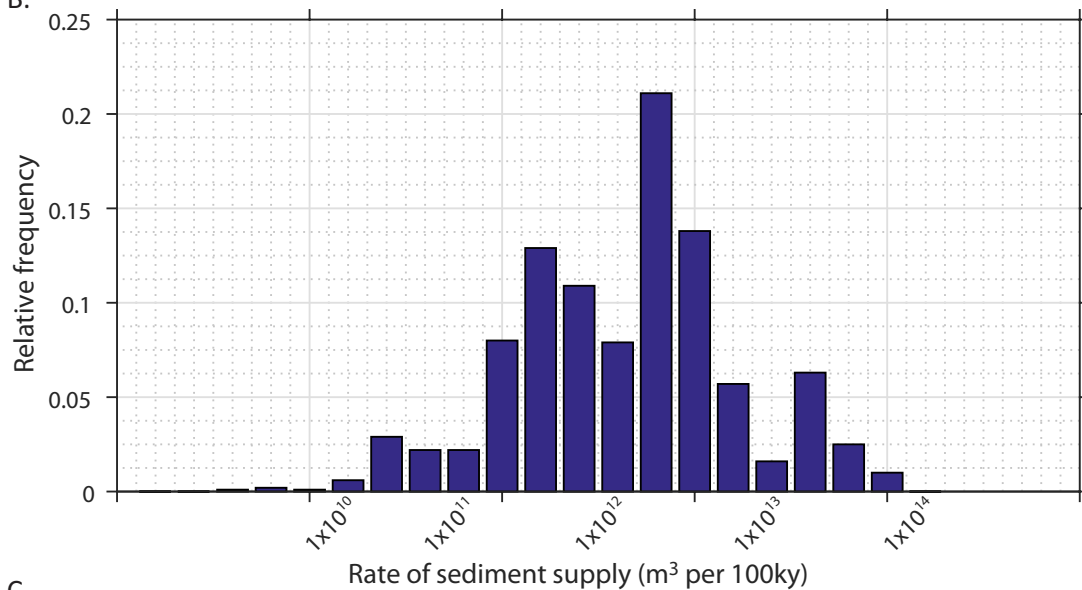
Figure 7. A subset of the stratal-control area with the outcrop stratal-control trajectory plotted. The outcrop trajectory is more complex than the hypothetical examples (Fig. 5), reflecting numerous changes in sediment supply and variable rate of relative sea-level change, and leading to a rather convoluted looping topology. Plotting the trajectory suggests that overall it is more accommodation dominated than supply dominated, and the ASTRR value of 72.91 supports this contention.



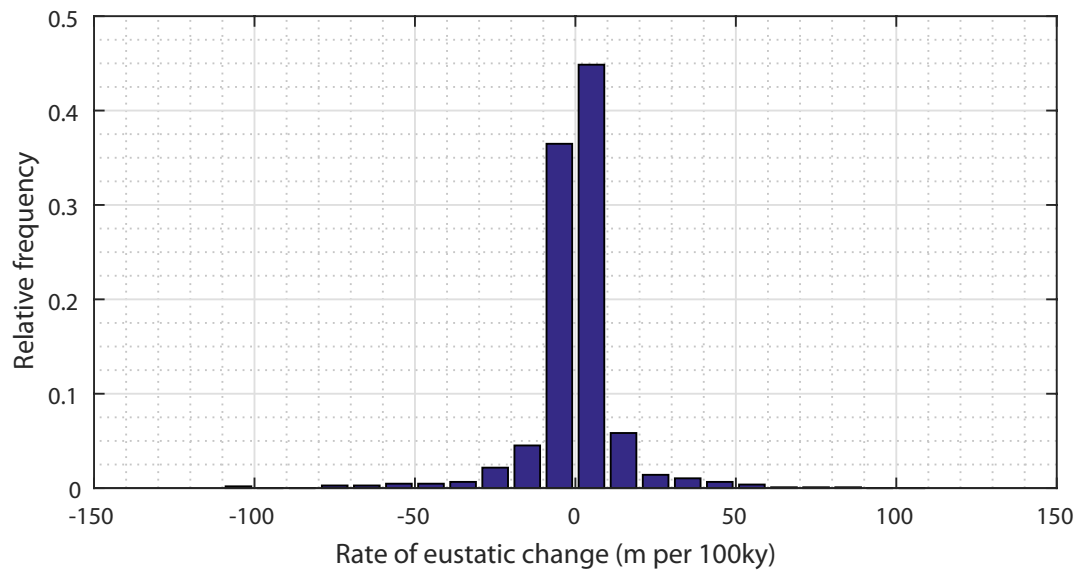
A.



B.



C.



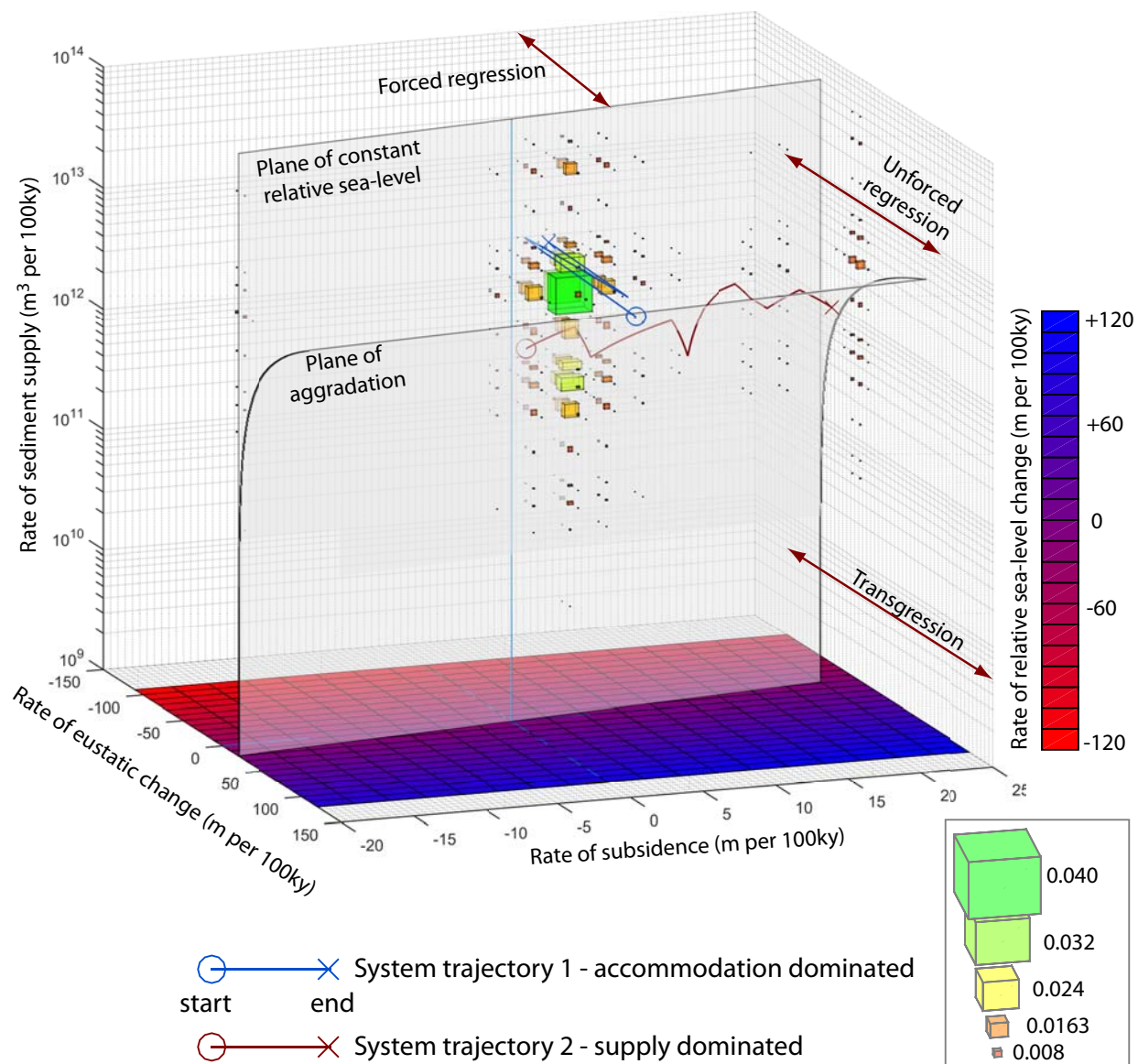


Figure 3

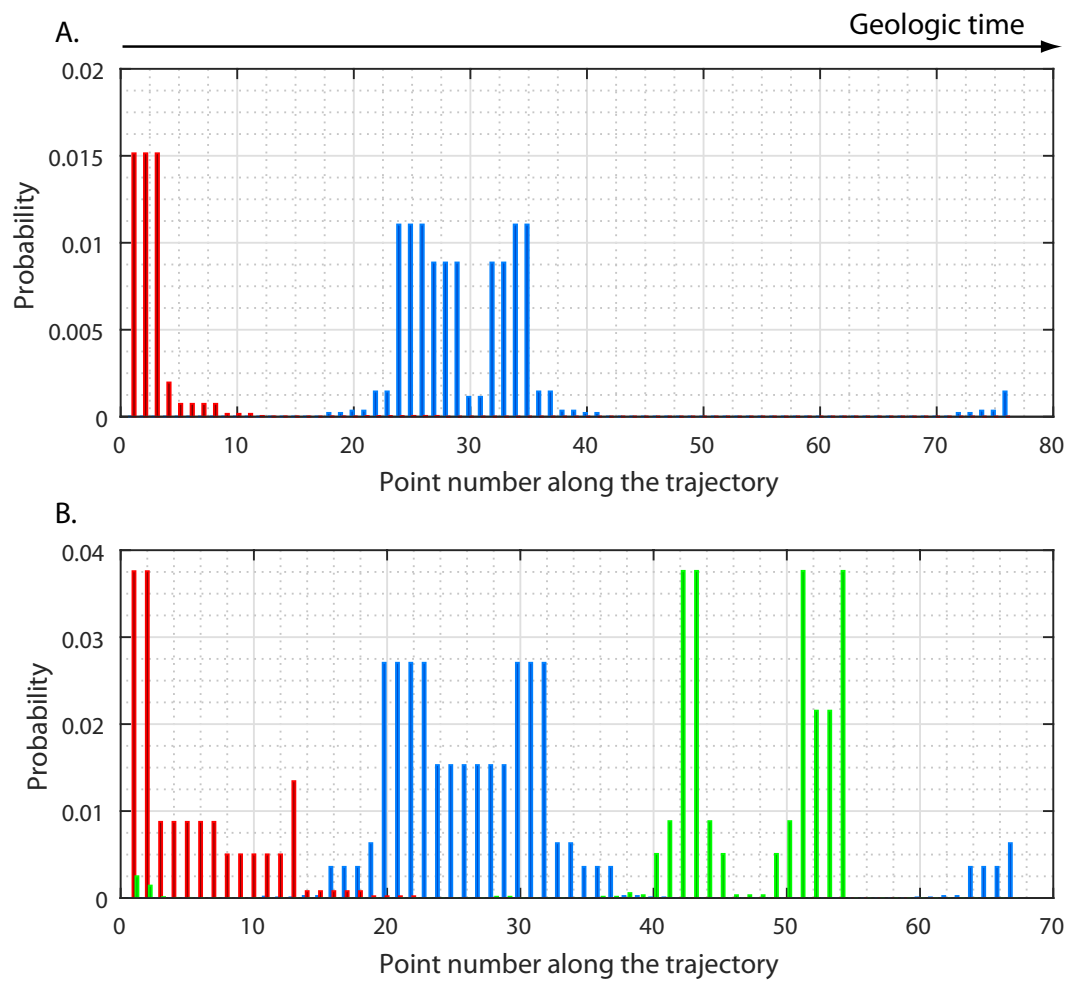
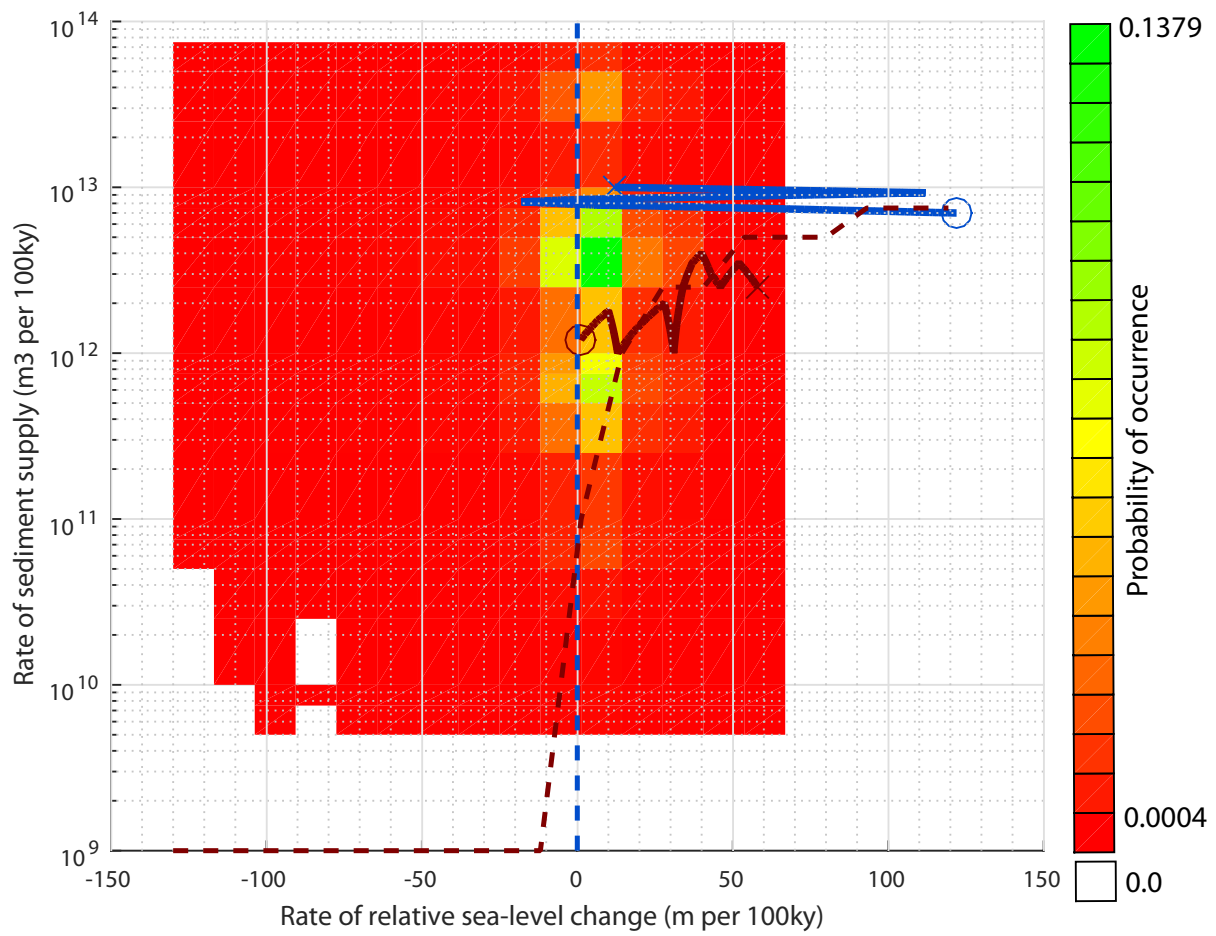


Figure 4



- Line of constant relative sea-level
- Line of aggradation
- — × System trajectory 1 - accommodation dominated
start end
- — × System trajectory 2 - supply dominated

Figure 5

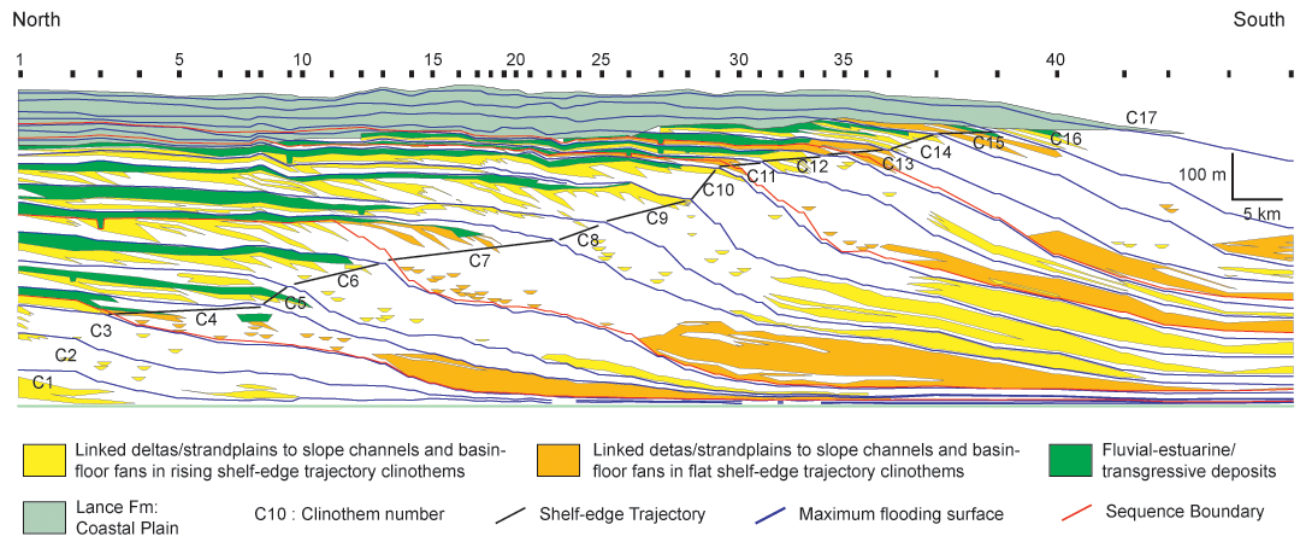


Figure 6

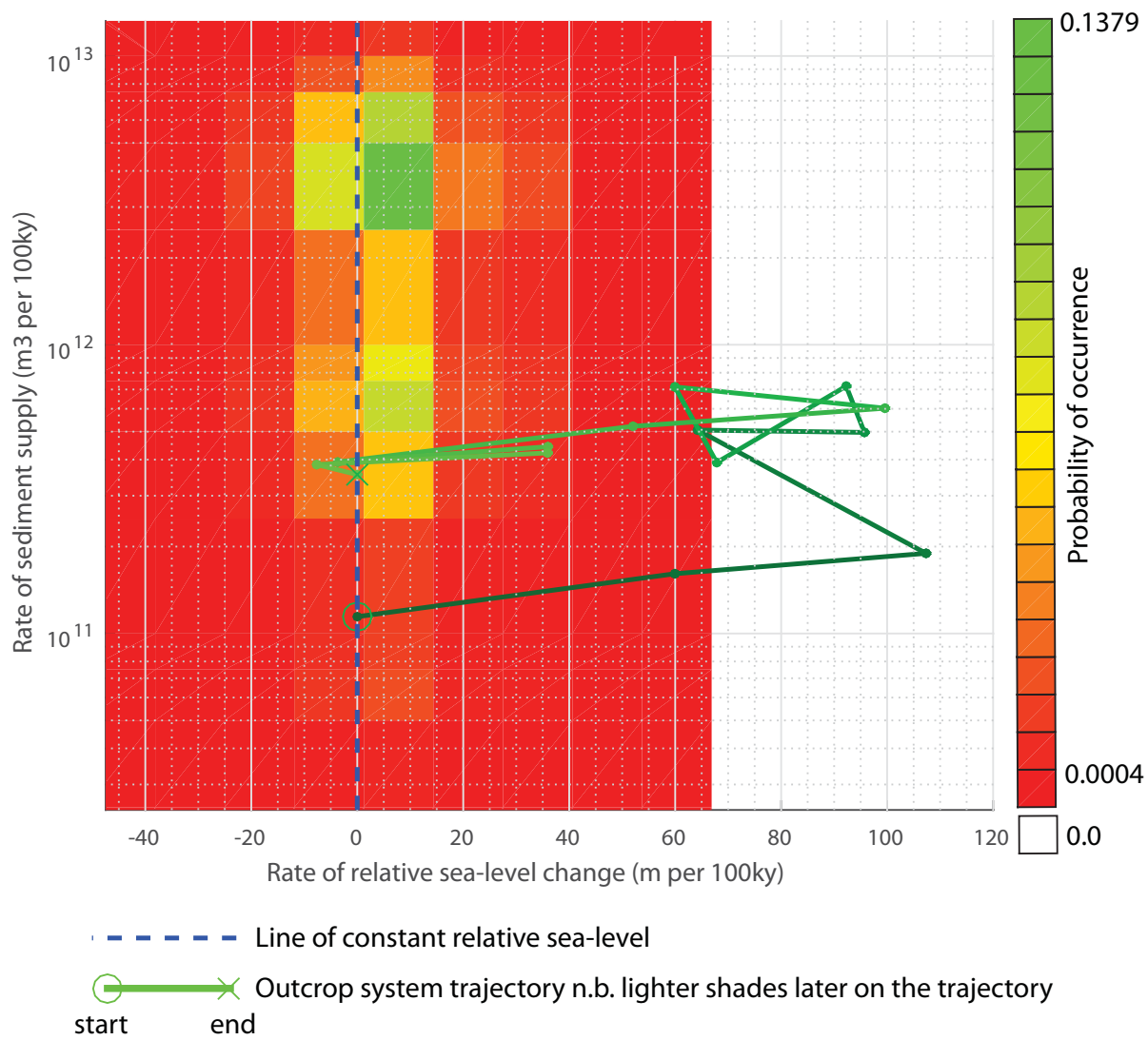


Figure 7